TABLE I. Calculational equations for the uniaxial stress data used with the hydrostatic pressure data to obtain the third-order elastic constants of single-crystal columbium. The relation numbers refer to the equations in order in Tables I–III of Ref. 26. The relations at the bottom can be derived from the equations of Ref. 26. The values of the adiabatic second-order elastic constants of columbium used to calculate the constants in these equations are the "best" values listed in Table II. The isothermal elastic constants $C_{11}^T$ and $C_{12}^T$ needed were determined from $C_{ij}^T = C_{ij}^S - 0.0161\times10^{12}$.

| Relation No. | Calculational equation |
|---|---|
| 10 | $C_{111}-3.8597C_{112}+2.8597C_{123}=(+8.7413m_{10}-2.2644)\times10^{12}$ |
| 11 | $C_{144}-1.8597C_{166}=(+4.3706m_{11}+1.6243)\times10^{12}$ |
| 13 | $C_{144}-0.0754C_{166}+8.2754C_{456}=(-4.7004m_{13}-2.9182)\times10^{12}$ |
| 14 | $C_{144}-0.0754C_{166}-8.2754C_{456}=(-4.7004m_{14}+1.7822)\times10^{12}$ |
| 16 | $C_{111}-2.0754C_{112}+1.0754C_{123}=(-9.4007m_{16}-11.6339)\times10^{12}$ |
| 17 | $C_{144}-0.0754C_{166}-8.2754C_{456}=(-4.7004m_{17}+0.6109)\times10^{12}$ |

where

$$m_{14}-m_{17}=(1-A)/2=0.2492,$$

$$A=2C_{44}/(C_{11}-C_{12})$$

$$C_{456}=C_{44}(m_{14}-m_{13}-1)$$

on sample 1 after irradiation to determine the TOEC for comparison with the previous determination.

There are several significant features of this study with regard to the determination of the TOEC. Assuming the $\Delta f$ effect observed is due to dislocation motion or rearrangement, it is apparent that this can occur at stresses well below the yield point of this material, and must be prevented in TOEC studies. The magnitude of the time-independent $\Delta f$ associated solely with lattice anharmonicity in this study ranged from 2 to 35 kHz for a stress of 4800 psi so the 0.5 kHz time-dependent $\Delta f$ would be significant on some runs. The reason for the different behavior of the two apparently identical samples is not known but may be due to slight differences in impurity levels or in cold-work during preparation, both of which might be methods for controlling the dislocation effect. Other methods for controlling this effect suggested by the dislocation study are preloading or load cycling the sample prior to use, irradiating it, or simply working at lower stresses. These three methods comprise the principle differences in procedure for the three TOEC determinations of the two samples of this study, i.e., (1) preloading of sample 1; (2) irradiating sample 1; (3) using low stress on sample 2.

There is another possible contribution of the dislocations to the change in frequency with applied uniaxial load besides the time-dependent frequency change observed at the higher values of load. This is due to the increase in dislocation loop length as the dislocations bow out with applied stress before breakaway occurs. This would cause an apparent reduction in the second-order dynamic elastic moduli, the magnitude of which would depend upon the initial loop length. It is probable that the difference between the behavior of the two samples in the dislocation study described above is due to a difference in loop length along with other factors, and that the loop length in sample 1 was different before and after irradiation. Therefore, if dislocation bowing in these samples causes an appreciable $\Delta f$, systematic differences in the three independent sets of measurements of the uniaxial stress dependences of the ultrasonic wave velocities should be apparent. It will be seen that no such differences were found and that this contribution of the dislocations to the measured frequency change is therefore small.

### D. Data Analysis

Analysis of the data to obtain values for the TOEC of columbium was done using the relations given by Thurston and Brugger[26] in their Tables I–III for cubic single crystals and IV for an isotropic medium. These relations will not be repeated here but will be referred to by numbers 1–17 for the single-crystal relations in the order in which they are given, and by 1′–5′ for the isotropic medium relations. These equations relate the stress derivatives of the second-order elastic constants evaluated at zero stress in terms of combinations of second-order and third-order elastic constants. Since these stress derivatives are independent of pressure within the accuracy of the measurements, they are determined by the experimental slopes of the null frequency vs load plots, i.e.,

$$m_n\equiv[(\partial/\partial p)(\rho_0 w^2)_n]_T\,|_{p=0}$$
$$=[F(C_{ij})_0/\Delta p][2\Delta f/f_0+(\Delta f/f_0)^2]_n, \quad (1)$$

where $\Delta f$ is the observed change in frequency for a total pressure or stress change $\Delta p$, and $F(C_{ij})$ is the combination of second-order elastic constants for the elastic wave mode associated with relation $n$ in Tables I–IV of Ref. 26. Since the largest $\Delta f/f_0$ value observed was $\sim10^{-3}$, the $(\Delta f/f_0)^2$ term can be ignored resulting

[26] R. N. Thurston and K. Brugger, Phys. Rev. **133**, A1604 (1964).